## Amendments to the Claims

The listing of claims will replace all prior versions, and listings of claims in the application.

- 1. (Previously Presented) An angle rotator for rotating an arbitrary input complex number to produce a rotated complex number according to an input angle  $\theta$ , said angle rotator comprising:
- a memory that stores a  $\sin\theta_M$  value and a  $\cos\theta_M$  value, wherein  $\theta_M$  is a coarse approximation to said input angle  $\theta$ ;
- a first digital circuit that performs a coarse rotation on said arbitrary input complex number based on said  $\sin\theta_M$  value and said  $\cos\theta_M$  value, resulting in an intermediate complex number;
- a fine adjustment circuit that generates a fine adjustment value based on a  $\theta_L$  value, wherein  $\theta_L$ =  $\theta$   $\theta_M$ ; and
- a second digital circuit that performs a fine rotation on said intermediate complex number based on said fine adjustment value, resulting in the rotated complex number.
- 2. (Previously Presented) The angle rotator of claim 1, wherein said fine adjustment value is  $(1 \theta_L^2/2)$  and wherein this fine adjustment value is produced by either a two's complement negation of  $\theta_L^2/2$  or a one's complement negation of  $\theta_L^2/2$ .

- 3. (Original) The angle rotator of claim 1, wherein said first digital circuit is a butterfly circuit having a plurality of multipliers that multiply said input complex number by said  $\sin \theta_M$  value and said  $\cos \theta_M$  value.
- 4. (Original) The angle rotator of claim 1, wherein said second digital circuit is a butterfly circuit having a plurality of multipliers that multiply said intermediate complex number by said fine adjustment value.

## 5-7. (Cancelled)

- 8. (Previously Presented) The angle rotator of claim 1, wherein said memory is indexed by  $\theta_{M.}$
- 9. (Previously Presented) An angle rotator for rotating an arbitrary input complex number to produce a rotated complex number according to an input angle  $\theta$ , said angle rotator comprising:

a memory that stores one or more values that are indexed by a most significant word (MSW) of said input angle, including a first value that is an approximation of a sin  $\theta_M$  value, and a second value that is an approximation of a cos  $\theta_M$  value, wherein  $\theta_M$  is a radian angle that corresponds to said MSW of the input angle, and

one or more error values that represent one or more quantization errors associated with at least one of said first value and said second value;

a first digital circuit that performs a coarse rotation on said arbitrary input complex number based on said first value and said second value, resulting in an intermediate complex number; and

a second digital circuit that performs a fine rotation on said intermediate complex number based on said one or more error values, resulting in the rotated complex number.

- 10. (Original) The angle rotator of claim 9, wherein said first digital circuit is a butterfly circuit.
- 11. (Original) The angle rotator of claim 10, wherein said butterfly circuit includes a plurality of multipliers that multiply said input complex number by said first value and said second value.
- 12. (Previously Presented) The angle rotator of claim 13, wherein said quantization error reflects a finite memory storage for said first value.
- 13. (Original) The angle rotator of claim 9, wherein said first value includes a memory quantization error relative to said  $\sin \theta_M$  value.
- 14. (Previously Presented) The angle rotator of claim 9, wherein said first value is a binary n-bit approximation of said  $\sin \theta_M$  value, wherein n is a bit storage capacity for said first value in said memory.

- 15. (Original) The angle rotator of claim 14, wherein said bit storage capacity is N/3 + 1 bits, wherein N is a number of bits that represent a real part of said input complex number.
- 16. (Previously Presented) An angle rotator for rotating an input complex number to produce a rotated complex number according to an input angle  $\theta$ , said angle rotator comprising:

a memory that stores one or more values that are indexed by a most significant word (MSW) of said input angle, including a first value that is an approximation of a sin  $\theta_M$  value, and a second value that is an approximation of a cos  $\theta_M$  value, wherein  $\theta_M$  is a radian angle that corresponds to said MSW of the input angle, and one or more error values that represent one or more quantization errors associated with at least one of said first value and said second value;

a first digital circuit that performs a coarse rotation on said input complex number based on said first value and said second value, resulting in an intermediate complex number; and

a second digital circuit that performs a fine rotation on said intermediate complex number based on said one or more error values, resulting in the rotated complex number;

wherein  $\theta_1$  is an arcsin of said first value, and wherein said one or more error values include a first error value that is a difference between said second value and said  $\cos \theta_1$ .

17. (Original) The angle rotator of claim 16, wherein said first error value is represented by as defined by the following equation:

$$\delta_{[\cos heta_{\scriptscriptstyle{\mathbf{i}}}]}$$

$$\frac{1}{\sqrt{\left[\cos\theta_{1}\right]^{2}+\left(\sin\theta_{1}\right)^{2}}}=1+\delta_{\left[\cos\theta_{1}\right]}.$$

- 18. (Previously Presented) The angle rotator of claim 16, wherein said one or more error values include a second error value that represents ( $\theta_{M}$   $\theta_{m}$ ), wherein  $\theta_{m}$  = arctan ( $\sin \theta_{1}$ / second value).
- 19. (Original) The angle rotator of claim 18, further comprising an adder that adds said second error value to  $\theta_L$  to produce a  $\theta_l$  value, wherein  $\theta_L$  is a radian angle associated with a least significant word (LSW) of said input angle.
- 20. (Original) The angle rotator of claim 19, wherein said angle rotator further comprises a fine adjustment circuit coupled to said second digital circuit, wherein said fine adjustment circuit generates a fine adjustment value based on  $\theta_l$  and said first error value.
- 21. (Original) The angle rotator of claim 20, wherein said fine adjustment value controls said fine angle rotation in said second digital circuit.
- 22. (Previously Presented) The angle rotator of claim 20, wherein said fine adjustment value is approximately: first error value minus  $(\frac{1}{2} \cdot \theta_l^2)$ .

- 23. (Original) The angle rotator of claim 20, wherein said second digital circuit includes a plurality of multipliers.
- 24. (Original) The angle rotator of claim 23, wherein said plurality of multipliers multiply said intermediate complex number by said  $\theta_l$  value.
- 25. (Original) The angle rotator of claim 23, wherein said plurality of multipliers multiply said intermediate complex number by said fine adjustment value.
- 26. (Previously Presented) An angle rotator for rotating an input complex number to produce a rotated complex number according to an input angle, said angle rotator comprising:
- a memory that stores one or more values indexed by a most significant word (MSW) of said input angle, including
- a first value that is an approximation of a tan  $\theta_M$  value, and a second value that is an approximation of a cos  $\theta_M$  value, wherein  $\theta_M$  is a radian angle that corresponds to said MSW of the input angle, and

one or more error values that represent one or more quantization errors associated with at least one of said first value and said second value;

a first digital circuit that rotates said input complex number based on said first value, resulting in an intermediate complex number; and

a second digital circuit that rotates said intermediate complex number so as to produce at least one part of the rotated complex number, based on said one or more error values and said second value, resulting in the rotated complex number.

- 27. (Previously Presented) The angle rotator of claim 26, wherein  $\theta_m$  is an arctan of said first value, wherein said one or more error values include a first error value associated with a value of  $\cos\theta_m$  minus said second value.
- 28. (Previously Presented) The angle rotator of claim 27, wherein said one or more error values include a second error value that represents  $\theta_M$   $\theta_m$ .
- 29. (Original) The angle rotator of claim 28, further comprising an adder that adds said second error value to  $\theta_L$  to produce a  $\theta_l$  value, wherein  $\theta_L$  is a radian angle associated with a least significant word (LSW) of said input angle.
- 30. (Previously Presented) The angle rotator of claim 29, wherein said angle rotator further comprises a fine adjustment circuit coupled to said second digital circuit, wherein said fine adjustment circuit generates a fine adjustment value based on  $\theta_l$  and said first error value.
- 31. (Original) The angle rotator of claim 30, wherein said fine adjustment value controls said fine angle rotation in said second digital circuit.

- 32. (Original) The angle rotator of claim 30, wherein said second digital circuit includes a plurality of multipliers.
- 33. (Original) The angle rotator of claim 32, wherein said plurality of multipliers multiply said intermediate complex number by said  $\theta_l$  value.
- 34. (Original) The angle rotator of claim 32, wherein said plurality of multipliers multiply said intermediate complex number by said fine adjustment value.
- 35. (Currently Amended) In a digital device, a method of rotating an arbitrary input complex number according to a representation of an input angle  $\theta$ , the method comprising the steps of:
- (1) receiving a digital signal, wherein a component of said digital signal is represented by the arbitrary input complex number;
- (2) determining a first value that is an approximation of  $\sin \theta_M$ , and determining a second value that is an approximation of  $\cos \theta_M$ , wherein  $\theta_M$  is a radian angle that corresponds to a most significant word (MSW) of the representation of the input angle  $\theta$ ; and
- (3) rotating said arbitrary input complex number in a complex plane based on said first value and said second value to generate a rotated complex number;
- (4) processing a signal by the digital device, whereby wherein said rotated complex number is used to generate said signal data processed by said digital device.

- 36. (Original) The method of claim 35, wherein said step of determining comprises the step of retrieving said first value and said second value from a memory.
- 37. (Currently Amended) The method of claim 35, further comprising the step of:
- (5) (4) determining a first error value that represents a difference between said second value and  $\cos \theta_1$ , where  $\theta_1$  is an arcsin of said first value.
- 38. (Currently Amended) The method of claim 37, further comprising the step of:
- (6) (5) rotating said rotated complex number in said complex plane to generate a second rotated complex number based on said first error value.
- 39. (Currently Amended) The method of claim 37, further comprising the step of:
- (6) (5) determining a second error value that represents ( $\theta_M$   $\theta_m$ ), wherein  $\theta_m$  = arctan (first value/second value).
- 40. (Currently Amended) The method of claim 39, further comprising the step of:
- (7) (6) adding said second error value to a  $\theta_L$  value to produce a  $\theta_l$  value, wherein  $\theta_L$  is a radian angle associated with a least significant word (LSW) of said input angle  $\theta$ .

- 41. (Currently Amended) The method of claim 40, further comprising the step of:
- (8) (7) generating a fine adjustment value based on said  $\theta_l$  value and said first error value.
- 42. (Previously Presented) The method of claim 41, wherein said fine adjustment value is approximately:

first error value minus  $(\frac{1}{2} \cdot \theta_1^2)$ .

- 43. (Currently Amended) The method of claim 41, further comprising the step of:
- (9) (8) rotating said rotated complex number according to said fine adjustment value and said  $\theta_l$  value.
- 44. (Currently Amended) The method of claim 43, wherein step (9) (8) comprises the steps of:
- (a) multiplying said rotated complex number by said fine adjustment value; and
  - (b) multiplying said rotated complex number by said  $\theta_l$  value.
- 45. (Currently Amended) In a digital device, a method of rotating an input complex number to produce at least one component of a rotated complex number according to an input angle  $\theta$ , the method comprising the steps of:

- (1) receiving a digital signal, wherein a component of said digital signal is represented by the input complex number;
- (2) determining a first value that is an approximation of  $\sin \theta_M$ , and determining a second value that is an approximation of  $\cos \theta_M$ , wherein  $\theta_M$  is a radian angle that corresponds to a most significant word (MSW) of the input angle  $\theta$ ;
- (3) rotating said input complex number in a complex plane based on said first value and said second value to generate an intermediate complex number;
- (4) determining one or more error values that represent one or more quantization errors, including the steps of
- (a) determining a first error value that represents a difference between said second value and  $\cos \theta_1$ , wherein  $\theta_1$  is an arcsin of said first value, and
- (b) determining a second error value that represents  $(\theta_M \theta_m)$ , wherein  $\theta_m$  = arctan (first value/ second value);
- (5) adding said second error value to a  $\theta_L$  value to produce a  $\theta_l$  value, wherein  $\theta_L$  is a radian angle associated with a least significant word (LSW) of said input angle  $\theta$ ;
- (6) generating a fine adjustment value based on  $\theta_l$  and said first error value; and
- (7) rotating said intermediate complex number in said complex plane to generate at least one component of the rotated complex number based on said  $\theta_l$  value and said fine adjustment value; and

- (8) processing a signal by the digital device, whereby wherein at least one component of the rotated complex number is used to generate said signal data processed by the digital device.
- 46. (Previously Presented) An angle rotator for rotating an input complex number to produce an output representing a single coordinate of a rotated complex number according to an input angle  $\theta$ , said angle rotator comprising:
- a memory that stores a first value representing  $\tan \theta_M$  and a second value representing an approximation of  $\cos \theta_M$ , wherein  $\theta_M$  is an approximation of said input angle  $\theta$ ;
- a first digital circuit that performs a first rotation on said input complex number based on said first value, resulting in an intermediate complex number;

means for generating a fine adjustment value;

a second digital circuit that performs a second rotation on said intermediate complex number based on said fine adjustment value to produce an output complex number; and

a scaling circuit that scales said output complex number using said second value to generate the single coordinate output.

47. (Previously Presented) The angle rotator of claim 46, wherein said fine adjustment value is based on a value of  $\theta_l$ , where  $\theta_l = \theta - \theta_m$ .

48. (Previously Presented) An angle rotator for a direct digital frequency synthesizer, for rotating a selected point in the complex plane according to an input angle θ to generate an output representing a rotated complex number, said angle rotator comprising:

a memory that stores a value representing  $\sin \theta_1$  and a value approximating  $\cos \theta_1$ , where  $\theta_1$  is an approximation of said input angle  $\theta$ ;

a first digital circuit that obtains said value representing  $\sin\theta_1$  from said memory using a value  $\theta_M$  to address said memory, where  $\theta_M$  is an approximation of said input angle  $\theta_3$ ;

means for generating a fine adjustment value; and

a second digital circuit that performs a rotation of a point in the complex plane whose coordinates are based on  $\sin\theta_1$  and said value approximating  $\cos\theta_1$  based on said fine adjustment value, to produce the output representing the rotated complex number.

- 49. (Previously Presented) The angle rotator of claim 48 wherein said fine adjustment value is based on a  $\theta_l$  value, wherein  $\theta_l = \theta \theta_m$ , where  $\theta_m = \arctan(\sin \theta_1 / \cos \theta_1)$  and  $[\cos \theta_1]$  is an approximation of  $\cos \theta_1$ .
- 50. (Previously Presented) An angle rotator for a direct digital frequency synthesizer, for rotating a selected point in the complex plane according to an input angle θ, to generate an output representing a single coordinate of a rotated complex number, said angle rotator comprising:

a memory that stores a value representing  $\sin \theta_1$  and a value approximating  $\cos \theta_1$ , where  $\theta_1$  is an approximation of said input angle  $\theta$ ;

a first digital circuit that obtains said value representing  $\sin\theta_1$  from said memory using a value based on  $\theta_M$  to address said memory, where  $\theta_M$  is an approximation of said input angle  $\theta$ ;

means for generating a fine adjustment value;

a second digital circuit that performs a rotation of a point in the complex plane whose coordinates are  $\sin\theta_1$  and said value approximating  $\cos\theta_1$ , based on said fine adjustment value, to produce one coordinate value of an output complex number; and a scaling circuit that scales said coordinate value of said output complex number using said value approximating  $\cos\theta_1$  to generate the single coordinate output.

- 51. (Previously Presented) The angle rotator of claim 50 wherein said fine adjustment value is based on a  $\theta_l$  value, wherein  $\theta_l = \theta \theta_m$ , where  $\theta_m = \arctan(\sin \theta_1 / [\cos \theta_1])$  and  $[\cos \theta_1]$  is said approximation of  $\cos \theta_1$ .
- 52. (Previously Presented) An angle rotator for a direct digital frequency synthesizer, for rotating a selected point in the complex plane according to an input angle  $\theta$ , to generate an output representing a single coordinate of a rotated complex number, said angle rotator comprising:

a memory that stores a value representing  $\tan \theta_1$  and a value approximating  $\cos \theta_1$ , where  $\theta_1$  is an approximation of said input angle  $\theta$ ;

a first digital circuit that obtains said value representing  $\tan \theta_1$  and said value approximating  $\cos \theta_1$  from said memory using a value representing  $\theta_M$  to address said memory, where  $\theta_M$  is an approximation of said input angle  $\theta$ ;

means for generating a fine adjustment value;

a second digital circuit that performs a rotation of the selected point in the complex plane by an angle approximating  $\theta_M$ , by using said value representing

 $\tan \theta_1$  and performing a second rotation of the resulting point in the complex plane, based on said fine adjustment value, to produce one coordinate of an output complex number; and

a scaling circuit that scales said output complex number, using said value approximating  $\cos \theta_1$ , to generate the single coordinate output.

53. (Currently Amended) A digital signal processing circuit for rotating an input complex number to produce a rotated complex number according to an input angle  $\theta$ , said circuit comprising:

a first digital circuit that generates a normalized input angle from a value representing the input angle  $\theta$ ;

a second digital circuit for stripping a plurality of most significant bits of said representation of the normalized input angle to obtain a temporary angle  $\tau$ ;

determining means for determining, from  $\tau$ , whether the normalized input angle represents an angle that is in an even or odd quadrant or octant; and

2's complement negate means for selectively negating the bits remaining after said stripping of most significant bits from the normalized input angle by selectively performing a 2's complement negate operation on said remaining bits;

wherein a resulting angle  $\varphi$  is equal to temporary angle  $\tau$  if said input angle is in an even <u>quadrant or</u> octant, and said resulting angle  $\varphi$  is equal to the 2's complement negation of temporary angle  $\tau$  if said input angle is in an odd <u>quadrant or</u> octant.

54. (Currently Amended) In a digital device, a A method of rotating an input complex number to produce an output representing a single coordinate of a rotated complex number according to an input angle  $\theta$ , comprising the steps of:

storing in a memory a value representing tan  $\theta_M$  and a value representing an approximation of  $\cos\theta_M$ , wherein  $\theta_M$  is an approximation of said input angle  $\theta$ ;

performing a first rotation on said input complex number based on said value representing tan  $\theta_M$ , resulting in an intermediate complex number;

generating a fine adjustment value;

performing a second rotation on said intermediate complex number based on said fine adjustment value, to produce an output complex number; and

scaling said output complex number, using said approximation of  $\cos\theta_M$ , to generate the single coordinate output; and

processing a signal by the digital device, wherein the single coordinate output is used during said processing.

- 55. (Previously Presented) The method of claim 54, wherein said fine adjustment value is based on a value of  $\theta_l$ , where  $\theta_l = \theta \theta_m$ .
- 56. (Currently Amended) A method for rotating a selected point in the complex plane in a direct digital frequency synthesizer, according to an input angle θ, to generate a direct digital frequency synthesizer an output signal component comprising at least one coordinate of representing a rotated complex number, comprising the steps of:

storing a value representing  $\sin\theta_1$  in a memory and a value approximating representing  $\cos\theta_1$  in a memory, where  $\theta_1$  is an approximation of said input angle  $\theta$ ;

obtaining said value representing  $\sin \theta_1$  from said memory using a value  $\theta_M$  to address said memory, where  $\theta_M$  is an approximation of said input angle  $\theta$ ; generating a fine adjustment value;

performing a rotation of a point in the complex plane whose coordinates are  $\sin \theta_1$  and said value approximating representing  $\cos \theta_1$ , based on said fine adjustment value, to produce the <u>direct digital frequency synthesizer</u> output <u>signal</u> component representing the rotated complex number.

57. (Previously Presented) The method of claim 56 wherein said fine adjustment value is based on a  $\theta_l$  value, wherein  $\theta_l = \theta - \theta_m$ , where  $\theta_m = \arctan(\sin \theta_1 / [\cos \theta_1])$  and  $[\cos \theta_1]$  is an approximation of  $\cos \theta_1$ .

Claims 58 and 59 (Cancelled).

60. (Currently Amended) A method for rotating a selected point in the complex plane in a direct digital frequency synthesizer, according to an input angle θ, to generate a direct digital frequency synthesizer an output signal component comprising representing a single coordinate of a rotated complex number, comprising the steps of:

storing in a memory a value representing  $\tan \theta_1$  and a value approximating  $\cos \theta_1$ , where  $\theta_1$  is an approximation of said input angle  $\theta$ ;

obtaining said value representing  $\tan \theta_1$  and said value approximating  $\cos \theta_1$  from said memory using a value representing  $\theta_M$  to address said memory, where  $\theta_M$  is an approximation of said input angle  $\theta$ ;

generating a fine adjustment value;

performing a rotation of the selected point in the complex plane by an angle approximating  $\theta_M$  by using said value representing  $\tan \theta_1$  and performing a second rotation of the resulting point in the complex plane, based on said fine adjustment value, to produce a value representing one coordinate of an output complex number; and

scaling said value representing one coordinate of said output complex number using said value approximating  $\cos \theta_1$  to generate the <u>direct digital frequency</u> synthesizer single coordinate output signal component.

61. (Currently Amended) In a digital circuit, processing digital data, a method for rotating an input complex number to produce a rotated complex number according to an input angle  $\theta$ , comprising the steps of:

generating a normalized input angle from a value representing the input angle  $\theta$ ;

stripping a plurality of most significant bits of said value representing the normalized input angle to obtain a temporary angle  $\tau$ ;

determining, from  $\tau$ , whether the normalized input angle represents an angle in an even or odd quadrant or octant; and

selectively performing a 2's complement negate operation on the bits remaining after said stripping of most significant bits from the normalized input angle,

wherein a resulting angle  $\varphi$  is equal to temporary angle  $\tau$  if said input angle is in an even <u>quadrant or</u> octant, and said resulting angle  $\varphi$  is equal to the 2's complement negation of temporary angle  $\tau$  if said input angle is in an odd <u>quadrant or</u> octant; and

processing a signal by the digital circuit, wherein the rotated complex number is used during said processing.

Claims 62-65 (Cancelled).

- 66. (Previously Presented) The angle rotator of claim 1, wherein said first digital circuit processes a digital binary representation of said arbitrary input complex number.
- 67. (Previously Presented) The angle rotator of claim 9, wherein said first digital circuit processes a digital binary representation of said arbitrary input complex number.
- 68. (Previously Presented) The method of claim 35 wherein said first and second value are represented and processed in digital binary form.

- 69. (Previously Presented) The angle rotator of claim 48, wherein the precision of the value approximating  $\cos \theta_1$  is at least 50% greater than the precision of the value representing  $\sin \theta_1$ .
- 70. (Previously Presented) The method of claim 56, wherein the precision of the value representing  $\cos\theta_1$  is at least 50% greater than the precision of the value representing  $\sin\theta_1$ .
- 71. (Previously Presented) The angle rotator of claim 1, wherein said first digital circuit processes a digital binary representation of said input complex number, with the real part and the imaginary part of the input complex number each having N bits, with N  $\geq$  9, and wherein said processing based on said  $\sin \theta_M$  and  $\cos \theta_M$  values employs binary representations of  $\sin \theta_M$  and  $\cos \theta_M$  in which the number of bits in the representation of the  $\cos \theta_M$  value is at least  $1.5 \times K$ , where K is the number of bits used in the representation of the  $\sin \theta_M$  value.
- 72. (Previously Presented) The angle rotator of claim 9, wherein said first digital circuit processes a digital binary representation of said input complex number, wherein said first value has a precision of K bits and said second value has a precision of at least  $1.5 \times K$  bits.

- 73. (Previously Presented) The method of claim 35, wherein said first and second value are represented and processed in digital binary form with the precision of the second value being at least 50% greater than that of the first value.
- 74. (New) The method of claim 45, wherein said first and second value are represented and processed in digital binary form with the precision of the second value being at least 50% greater than that of the first value.
- 75. (New) The angle rotator of claim 16, wherein said first digital circuit processes a digital binary representation of said input complex number, with the real part and the imaginary part of the input complex number each having N bits, with  $N \ge 9$ , and wherein said processing based on said  $\sin \theta_M$  and  $\cos \theta_M$  values employs binary representations of  $\sin \theta_M$  and  $\cos \theta_M$  in which the number of bits in the representation of the  $\cos \theta_M$  value is at least  $1.5 \times K$ , where K is the number of bits used in the representation of the  $\sin \theta_M$  value.
- 76. (New) The angle rotator of claim 26, wherein said first digital circuit processes a digital binary representation of said input complex number, wherein said first value has a precision of K bits and said second value has a precision of at least  $1.5 \times K$  bits.

- 77. (New) The angle rotator of claim 46, wherein said first digital circuit processes a digital binary representation of said input complex number, wherein said first value has a precision of K bits and said second value has a precision of at least  $1.5 \times K$  bits.
- 78. (New) The angle rotator of claim 52, wherein said value representing  $\tan \theta_1$  has a precision of K bits and said value approximating  $\cos \theta_1$  has a precision of at least  $1.5 \times K$  bits.
- 79. (New) The method of claim 54, wherein said value representing  $\tan \theta_M$  has a precision of K bits and said value representing an approximation of  $\cos \theta_M$  has a precision of at least  $1.5 \times K$  bits.
- 80. (New) The method of claim 60, wherein said value representing  $\tan \theta_1$  has a precision of K bits and said value representing an approximation of  $\cos \theta_1$  has a precision of at least  $1.5 \times K$  bits.